Then d is found to equal, we aver,
Square root of 3 plus one times half of a. [$\frac{1}{2}a(\sqrt{3}+1)$]
Equating d's two different values, weigh
Results, and see *square root of 3 plus 2.* ($\sqrt{3}+2$]
Of b to a the ratio, come to view.
This ratio known as t; then find it true
That of diameters of any two
Contiguous spheres within the corner's space
The ratio is the self-same t, a case
Of beauty mathematical. We now
From demonstrations rest the knitted brow,
And the diameters of all the spheres
Announce. Within the problem it appears
That the diameter of marble sphere
Is just *one inch*. From ratio t 't is clear
That iron sphere's diameter is t;
Then that of copper sphere *t squared* must be;
Whence that of silver sphere is *cube of t*;
And the diameter of golden sphere
As *t involved to fourth power* must appear.
Three hundred seventy-three as hundredths is
The value of the ratio t; and this [t=3.73+]
To powers second, third and fourth involved,
Yields as results, approximately solved, [13.92+]
Nine tenths to thirteen added, fifty-two, [51.98+]
And ninety-seven multiplied by two. [193.99+]

The golden sphere is wondrous as to size;
And in regard to value as a prize,
Not all our country's golden output coined
In her existing years, together joined,
Could purchase its great worth. In numbers round,
Eight hundred four of millions will be found
The dollars that in purest gold abound
In this great sphere. Besides it would confound
The efforts of the mind that should aspire
To measure the extent of thinnest wire
To which there might be drawn this golden mass.
From Earth to Sun this slender thread could pass,
Return again to Earth, and eight times more
The circuit make, and still have end galore
To stretch from Earth to Moon strands eighteen score.

ALGEBRA.

104. Prize Problem. $2.50 for the best solution.

Compute to three decimal places each of the roots of the equation $x^2+y=2$, $x+y^2=6$.

I. Solution by AGNES E. SCHEFFER, Hagerstown, Md.

From the first of these equations we have $y=2-x^2$, and substituting this in the second, we have $x^4-4x^2+x-2=0$.

Factoring we have $x^2(x^2-4)+(x-2)=0$, or $(x-2)(x^3+2x^2+1)=0$.

$\therefore$ $x-2=0$, and $x^3+2x^2+1=0$.

From the former we obtain $x=2$ and then its simultaneous value $y=-2$, obtained from the first of the original equations.

By solving the cubic equation $x^3+2x^2+1=0$, we obtain three more roots for x.

Putting $x=z-\frac{2}{3}$, we get $z^3-\frac{4}{3}z+\frac{43}{27}=0$, and employing Cardon's formula we have

$$y=\sqrt[3]{\left[-\tfrac{43}{27}+\tfrac{1}{54}\sqrt{(1593)}\right]}+\sqrt[3]{\left[-\tfrac{43}{54}-\tfrac{1}{54}\sqrt{(1593)}\right]}, \text{ or}$$

$$z=\tfrac{1}{3}\left\{\sqrt[3]{\left[-\tfrac{43}{2}+\tfrac{1}{2}\sqrt{(1593)}\right]}+\sqrt[3]{\left[-\tfrac{43}{2}-\tfrac{1}{2}\sqrt{(1593)}\right]}\right\}, \text{ and}$$

$$x=-\tfrac{2}{3}+\tfrac{1}{3}\left\{\sqrt[3]{\left[-\tfrac{43}{2}+\tfrac{1}{2}\sqrt{(1593)}\right]}+\sqrt[3]{\left[-\tfrac{43}{2}-\tfrac{1}{2}\sqrt{(1593)}\right]}\right\}=-2.205569.$$

Dividing x^3+2x^2+1 by $x+2.205569$ we get $x^2-205569x+.4533966$, and now the equation $x^2-.205569x+.4533966=0$ furnishes us the other two roots of x, viz: $x=.102784\pm.665456\sqrt{-1}$.

The simultaneous values of y we obtain from the equation $x^2+y=2$, viz: $y=2-x^2$.

Thus, we find the following four sets of the simultaneous values of x and y:

$$\begin{array}{l|l|l} x=2 & x=-2.205569 & x=.102784\pm.665456\sqrt{-1} \\ y=-2 & y=-2.864534 & y=2\ 432267\mp.136796\sqrt{-1} \end{array}$$

II. Solution by J. W. YOUNG, Fellow and Assistant in Mathemetics, Ohio State University, Columbus, O.

I. Solve (1) for y, substitute in (2) and obtain

$$x^4-4x^2+x-2=0.$$

In the application of Horner's method or by inspection we see that $2=x_1$ is a root. By dividing out this root we obtain for the equation giving the remaining roots

$$x^3+2x^2+1=0\ldots(3).$$

This equation has a pair of complex roots. Denoting the real root by x_2, and applying Horner's process, we find

$$x_2=-2.2055+.$$

Denoting the other roots x_3, x_4 by $\alpha\pm i\beta$ and observing that the sum of the roots of (3) equals -2, we have

$$2\alpha-2.2055=-2.$$

Whence $\alpha=0.1027+$.

Similarly, the product of the roots,

$$x_2(\alpha^2+\beta^2)=-1.$$

Whence $\beta=0.6654+$.

Hence the complex roots are x_3, $x_4=0.1027\pm i0.6654+$.

Collecting results and calculating the corresponding values of y, we have, as a complete solution of the original system.

$$x_1=2. \qquad\qquad y_1=-2.$$

$$x_2=-2.2055 \qquad\qquad y_2=-2.8642$$

$$\left.\begin{matrix}x_3\\x_4\end{matrix}\right\}=0.1027\pm i0.6654 \qquad \left.\begin{matrix}y_3\\y_4\end{matrix}\right\}=2.4323\mp i0.1367$$

II. Since the cubic giving the incommensurable roots has a pair of complex roots, Cardan's solution may be applied. In the cubic (3) put $x=z-\frac{2}{3}$, and obtain

$$z^3-\tfrac{12}{9}z+\tfrac{43}{27}=0\;\ldots\;(4).$$

Put $z=\sqrt[3]{p}+\sqrt[3]{q}$, and cube. Then

$$z^3-3\sqrt[3]{p}\sqrt[3]{q}z-(p+q)=0\;\ldots\;(5).$$

Comparing coefficients in (4) and (5), we have

$$p+q=-\tfrac{43}{27},\; pq=\tfrac{64}{729}.$$

Whence $p=-0.0572$ and $\sqrt[3]{p}=-0.3853$, $q=-1.5354$ and $\sqrt[3]{q}=-1.1536$.

Hence the real root of $(4)=\sqrt[3]{p}+\sqrt[3]{q}=-1.5389$ and $x_2=z-\frac{2}{3}=-1.5389-0.7666=-2.2055$ as before.

The complex roots are given by $w\sqrt[3]{p}+w^2\sqrt[3]{q}$ and $w^2\sqrt[3]{p}+w\sqrt[3]{q}$, where $w=\frac{1}{2}(-1\pm i\sqrt{3})$. Computing these roots by the above formulae, we obtain as before

$$x_3,\; x_4=0.1027\pm i0.6654.$$

III. In equation (3) substitute $x=\alpha+i\beta$, and obtain, after separating real and imaginary parts

$$\alpha^3+2\alpha^2+1-\beta^2(\beta\alpha+2)=0\;\ldots\;(6),$$

$$\beta(\beta^2-3\alpha^2-4\alpha)=0\;\ldots\;(7).$$

From (7), $\beta=0$,

$$\beta^2=3\alpha^2-4\alpha\;\ldots\;(8).$$

For $\beta=0$ (x real) we have then $\alpha^3+2\alpha^2+1=0$ (from (6)) giving the real root, $x_2=-2.2055$.

For $\beta^2=3\alpha^2-4\alpha$, we have from (6),

$$8\alpha^3+16\alpha^2+8\alpha-1=0.$$

Solving this we obtain $\alpha=0.1027$ as before. And from (8) $\beta=0.6654$.

IV. We may write (1) and (2) in the form

$$x^2-4=-(2+y)$$

$$x-2=4-y^2.$$

From which immediately $x=2$, $y=-2$.

Dividing we have $x+2=-\dfrac{1}{2-y}$, or $y=\dfrac{2x+5}{x+2}$.

Substitute in (1), and obtain $x^3+2x^2+1=0$. Solve this by any of the above methods.

V. Add (1) and (2) and complete squares. Then

$$x^2+x+\tfrac{1}{4}+y^2+y+\tfrac{1}{4}=8+\tfrac{2}{4} \text{ or } (x+\tfrac{1}{2})^2+(y+\tfrac{1}{2})^2=\tfrac{25}{4}+\tfrac{9}{4}.$$

Whence, from the four possible corresponding values of x and y, we may pick out one set which satisfies the original system, namely $x=2$, $y=-2$.

Also solved by *GEO. R. BERRY.*

Note. The donor of this prize has acted as judge on the merits of the several solutions, and his decision is that the two published solutions are of equal merit. In accordance with this decision, the prize money has been equally divided between Miss Scheffer and Mr. Young. We might say that there has been only one solution sent in to the prize problem in Mechanics. This solution is defective. The problem is, therefore, open to all our contributors for solution. EDITOR F.

105. Proposed by CHARLES E. MYERS, Canton, O.

Solve for x the following: $a^{\log(a^{x^2})}=m^{\log(m)}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.; J. K. ELLWOOD, A. M., Colfax School, Pittsburg, Pa.; J. SCHEFFER, A. M., Hagerstown, Md.; COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; W. F. SHAW, Austin, Tex.; and ELMER SCHUYLER, B. Sc., Professor of German and Mathematics, Boys' High School, Reading, Pa.

$a^{\log(a^{x^2})}=m^{\log m}$ may be written $a^{x^2\log a}=m^{\log m}$.

$\therefore\ x^2(\log a)^2=(\log m)^2$.

$\therefore\ x=\pm(\log m/\log a)$.

GEOMETRY.

130. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

If the points x, y, z divide the strokes $c-b$, $a-c$, $b-a$, in the same ratio r, and the triangles x, y, z and a, b, c are similar, either $r=1$ or both triangles are equilateral. [From Harkness and Morley's *Introduction to the Theory of Functions*, page 26].

Solution by the PROPOSER.

Let x, y and z denote the points, dividing $c-b$, $a-c$, and $b-a$, respectively, in the given ratio r.

Then $x=\dfrac{c+br}{1+r}$, $y=\dfrac{a+cr}{1+r}$, and $z=\dfrac{b+ar}{1+r}$.

The condition that a, b, c, and x, y, z form similar triangles is

$$\begin{vmatrix} a & x & 1 \\ b & y & 1 \\ c & z & 1 \end{vmatrix}=0\ldots.(1).$$